53. (a) The angular frequency is $\omega = 8.00\pi/2 = 4.00\pi$ rad/s, so the frequency is $f = \omega/2\pi = (4.00\pi \text{ rad/s})/2\pi = 2.00$ Hz.

(b) The angular wave number is $k = 2.00\pi/2 = 1.00\pi \text{ m}^{-1}$, so the wavelength is $\lambda = 2\pi/k = 2\pi/(1.00\pi \text{ m}^{-1}) = 2.00$ m.

(c) The wave speed is

$$v = \lambda f = (2.00\,\text{m})(2.00\,\text{Hz}) = 4.00 \text{ m/s}.$$

(d) We need to add two cosine functions. First convert them to sine functions using $\cos\alpha = \sin(\alpha + \pi/2)$, then apply

$$\cos\alpha + \cos\beta = \sin\left(\alpha + \frac{\pi}{2}\right) + \sin\left(\beta + \frac{\pi}{2}\right) = 2\sin\left(\frac{\alpha+\beta+\pi}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$
$$= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

Letting $\alpha = kx$ and $\beta = \omega t$, we find

$$y_m \cos(kx + \omega t) + y_m \cos(kx - \omega t) = 2y_m \cos(kx)\cos(\omega t).$$

Nodes occur where $\cos(kx) = 0$ or $kx = n\pi + \pi/2$, where n is an integer (including zero). Since $k = 1.0\pi \text{ m}^{-1}$, this means $x = \left(n + \tfrac{1}{2}\right)(1.00\,\text{m})$. Thus, the smallest value of x which corresponds to a node is $x = 0.500$ m (n=0).

(e) The second smallest value of x which corresponds to a node is $x = 1.50$ m (n=1).

(f) The third smallest value of x which corresponds to a node is $x = 2.50$ m (n=2).

(g) The displacement is a maximum where $\cos(kx) = \pm 1$. This means $kx = n\pi$, where n is an integer. Thus, $x = n(1.00$ m). The smallest value of x which corresponds to an anti-node (maximum) is $x = 0$ (n=0).

(h) The second smallest value of x which corresponds to an anti-node (maximum) is $x = 1.00$ m (n=1).

(i) The third smallest value of x which corresponds to an anti-node (maximum) is $x = 2.00$ m (n=2).